Stolarsky-3 Mean Labeling of Graphs

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Abstract; Let G = (V, E) be a graph with p vertices and q edges . G is said to be Stolarsky-3 Mean graph if each vertex x∈V is assigned distinct labels f(x) from 1,2,….,q+1 and each edge e=uv is assigned the distinct labels f(e=uv) = $\left\lceil\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rceil$ (or) $\left\lfloor\sqrt{\frac{f(u)^2+f(u)f(v)+f(v)^2}{3}}\right\rfloor$ then the resulting edge labels are distinct. In this case f is called a Stolarsky-3 Mean labeling of G and G is called a Stolarsky-3 Mean graph. In this paper we prove that PathP_n, Cycle C_n , Comb$P_n \odot k_1$, Ladder L_n ,Star $K_{1,n}$, Triangular Snake T_n , Quadrilateral Snake Q_n are Stolarsky -3 Mean graphs.

Keywords: Graph Labeling, Mean Labeling, Stolarsky-3 Mean Labeling

1. Introduction

The graph G = (V,E) considered here will be finite, simple and undirected. We follow Gallian**[1]** for all detailed survey of graph labeling and we refer Harary**[2]** for all other standard terminologies and notations. S.Somasundaram and R.Ponraj introduced the concept of "**Mean Labeling of Graphs" in 2004[3]** and S.Somasundaram and S.S. Sandhya introduced the concept of "**Harmonic Mean Labeling of graphs "in[4].** S.S. Sandhya, E.Ebin Raja Merely and S.Kavitha introduce a new type of Labeling called "**Stolarsky-3 Mean**

Labeling of Graphs"

We will give the following definitions and other information's which are helpful for our present investigation.

Definition 1.1: A walk in which all the vertices $u_1, u_2, \dots, u_n$ are distinct is called a path. It is denoted by P_n.

Definition 1.2: A closed path is called a cycle. A cycle on n vertices is denoted by C_n.

Definition 1.3: The graph obtained by adding a single pendant edge to each vertex of a path of n vertices is called a comb. It is denoted by $P_n \odot K_1$.

Definition 1.4: The Cartesian product of two graphs $G_1=(V_1, E_1)$ and $G_2=(V_2, E_2)$ is a graph G = (V, E) with V=$V_1 \times V_2$ and two vertices u=(u_1, u_2) and v=(v_1, v_2) are adjacent in $G_1 \times G_2$ whenever ($u_1 = v_1$ and u_2 is adjacent to v_2) or ($u_2 = v_2$ and u_1 is adjacent to v_1). It is denoted by $G_1 \times G_2$.

Definition 1.5: The Ladder graph L_n (n≥ 2) is the product graph $P_2 \times P_n$ which contains 2n vertices and 3n-2 edges.

Definition 1.6: A Triangular Snake T_n is obtained from a path $u_1, u_2, \dots, u_n$ by joining u_i and u_{i+1} to a new vertex v_i for 1≤ i ≤n-1. That is, every edge of a path is replaced by a triangleC_3.

Definition 1.7: A Quadrilateral snake Q_n is obtained from a path $u_1, u_2, \dots, u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i respectively and then joining $v_i$andw_i. That is, every edge of a path is replaced by a cycle C_4.

Definition 1.8: A bigraph (or a bipartite graph) is a graph whose vertex set V can be partitioned into two subsets V_1 andV_2 such that every edge of G joins a vertex of $V_1$to a vertex of V_2. (V_1, V_2) is called a bipartition of G. If further G contains every vertex of V_1 is joining to all the vertices of V_2then G is called a complete bigraph. It is denoted by $K_{m,n}$ where $|V_1| = m$ and $|V_2|$ =n.

Definition 1.9: A star is a complete bipartite graph$K_{1,n}$.

2. Main Results

Theorem 2.1: Any Path P_n is a Stolarsky-3 Mean graph.

Proof: Let $u_1, u_2, \dots, u_n$ be the vertices of the Path P_n whose length is n.

Define a function **f** : V(P_n)→{ 1,2,…., q+1} by

$$f(u_i) = i,\ 1 \le i \le n.$$

Then the edges are labeled as

$$f(u_i u_{i+1}) = i,\ 1 \le i \le n-1.$$

Thus we get distinct edge labels.

Hence Path P_n is Stolarsky-3 Mean graph.

Example 2.2: The Stolarsky-3 Mean labeling of P_7 is given below.

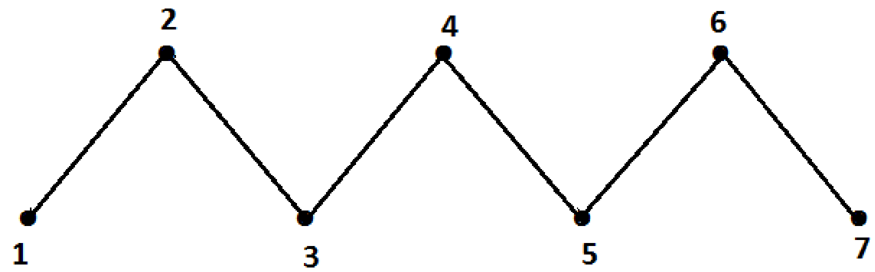


Figure: 1

Theorem 2.3: Any Cycle C_n is a Stolarsky-3 Mean graph.

Proof: Let $u_1, u_2, \dots, u_n$,u_1 be the cycle C_n whose length is n.

Define a function **f** : V(C_n)→{ 1,2,…., q+1} by

$$f(u_i) = i,\ 1 \le i \le n.$$

Then the edge labels are distinct.

Hence Cycle C_n is Stolarsky-3 Mean graph.

Example 2.4: The Stolarsky-3 Mean labeling of C_8 is given below.

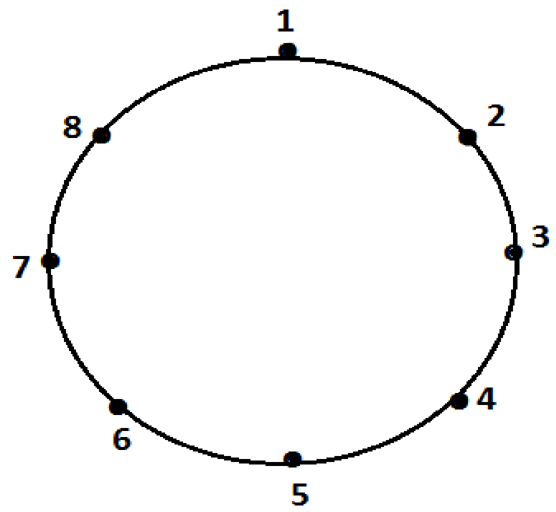


Figure: 2

Theorem 2.5: Comb $(P_n \Theta K_1)$ is a Stolarsky-3 Mean graph.

Proof: Let G be the Comb with vertices $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$.

Let P_n be the path $u_1, u_2, \dots, u_n$ and join a vertex u_i to v_i, $1\leq i \leq n$.

Define a function $f: V(G) \to \{1,2,\dots, q+1\}$ by

$f(u_i) = 2i-1$, $1\leq i \leq n$.

$f(v_i) = 2i$, $1\leq i \leq n$.

Then the edges are labeled as

$f(u_i u_{i+1}) = 2i$, $1\leq i \leq n-1$.

$f(u_i u_{i+1}) = 2i-1$, $1\leq i \leq n$.

Thus we get distinct edge labels.

Hence Comb $(P_n \Theta K_1)$ is a Stolarsky-3 Mean graph.

Example 2.6: Stolarsky-3 Mean Labeling of Comb obtained from P_5 is given below.

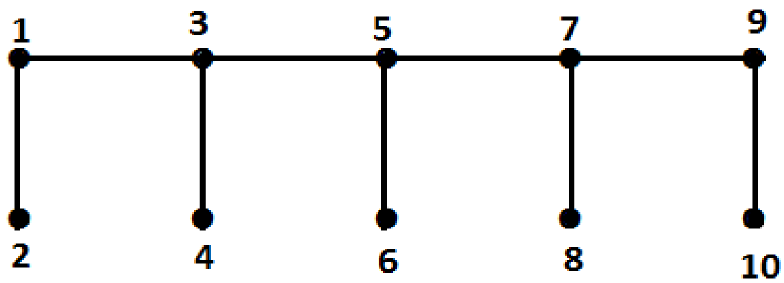


Figure:3

Theorem 2.7: The Ladder $L_n = P_2 \times P_n$ is a Stolarsky-3 Mean graph.

Proof: Let G be the Ladder graph with the vertices $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$.

Define a function $f: V(G) \to \{1,2,\dots, q+1\}$ by

$f(u_i) = 3i-2$, $1\leq i \leq n$.

$f(v_i) = 3i-1$, $1\leq i \leq n$.

Then the edges are labeled as

$f(u_i u_{i+1}) = 3i-1$, $1\leq i \leq n-1$.

$f(v_i v_{i+1}) = 3i$, $1\leq i \leq n-1$.

$f(u_i v_i) = 3i - 2, 1 \leq i \leq n.$

Thus we get distinct edge labels.

Hence Ladder $L_n = P_2 \times P_n$ is a Stolarsky-3 Mean graph

Example 2.8: The Stolarsky-3 Mean labeling of L_5 is given below.

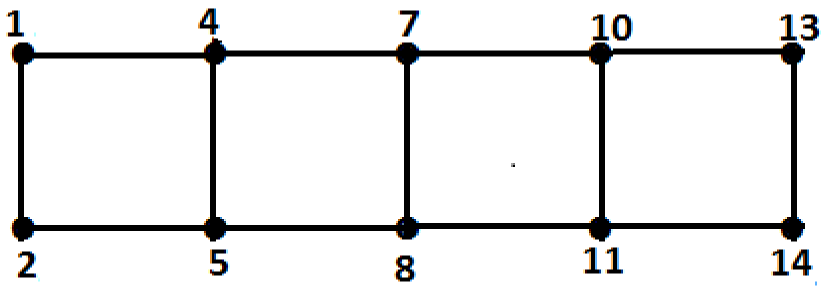


Figure: 4

Theorem 2.9: Any Triangular Snake T_n is a Stolarsky-3 Mean graph.

Proof: Let T_n be the Triangular snake graph with the vertices $u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_{n-1}$.

Consider a Path $u_1, u_2, \dots, u_n$. Join u_i *and* u_{i+1} to new vertex v_i, $1 \leq i \leq n-1$

Define a function $f : V(T_n) \to \{1,2,\dots, q+1\}$ by

$f(u_i) = 3i-2, 1 \leq i \leq n.$

$f(v_i) = 3i-1, 1 \leq i \leq n-1.$

Then the edges are labeled as

$f(u_i u_{i+1}) = 3i - 1, 1 \leq i \leq n-1.$

$f(u_i v_i) = 3i - 2, 1 \leq i \leq n-1.$

$f(v_i u_{i+1}) = 3i, 1 \leq i \leq n-1.$

Thus we get distinct edge labels.

Hence Triangular Snake graph T_n is a Stolarsky-3 Mean graph.

Example 2.10: The Stolarsky-3 Mean labeling of T_6 is given below.

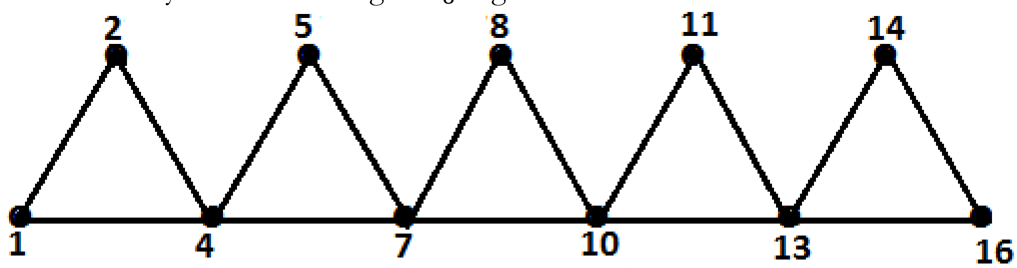


Figure: 5

Theorem 2.11: Any Quadrilateral Snake Q_n is a Stolarsky-3 Mean graph.

Proof: Let Q_n be the Quadrilateral Snake with the vertices $u_1, u_2, \dots, u_n$, $v_1, v_2, \dots, v_{n-1}$ *and* $w_1, w_2, \dots, w_{n-1}$. Consider a Path $u_1, u_2, \dots, u_n$. Join u_i *and* u_{i+1} to two new vertices v_i, and w_i $1 \leq i \leq n-1$.

Define a function $f : V(Q_n) \to \{1,2,\dots, q+1\}$ by

$f(u_i) = 4i-3, 1 \leq i \leq n.$

$f(v_i) = 4i-2, 1 \leq i \leq n-1.$

$f(w_i) = 4i-1, 1 \leq i \leq n-1.$

Then the edges are labeled as

$f(u_i u_{i+1}) = 4i - 1, 1 \leq i \leq n-1.$

$f(u_i v_i) = 4i - 3,\ 1 \leq i \leq n-1.$

$f(v_i u_{i+1}) = 4i,\ 1 \leq i \leq n-1.$

Thus we get distinct edge labels.

Hence Quadrilateral Snake Q_n is a Stolarsky-3 Mean graph.

Example 2.12: Stolarsky-3 Mean labeling of Q_5 is given below.

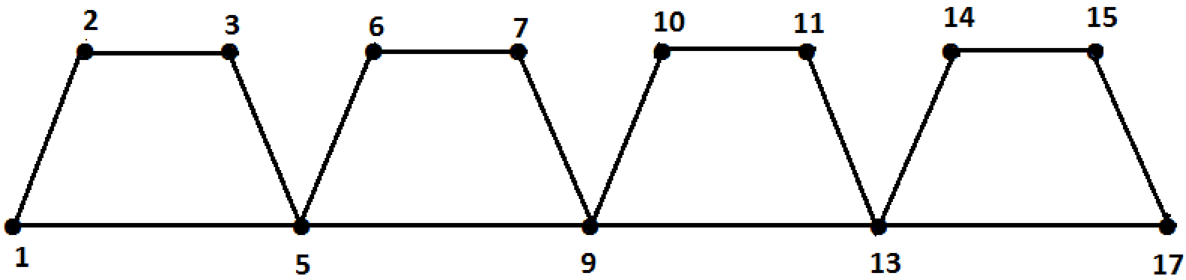


Figure:6

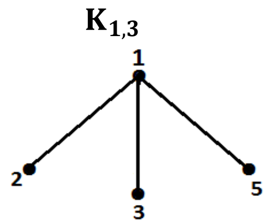
Theorem 2.13: $K_{1,n}$ is a Stolarsky-3 Mean graph if n ≤ 15.

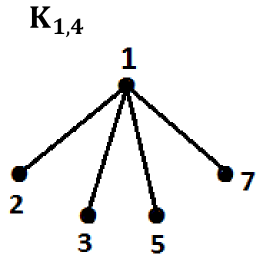
Proof: $K_{1,1}, K_{1,2}$ are Stolarsky-3 Mean graphs.

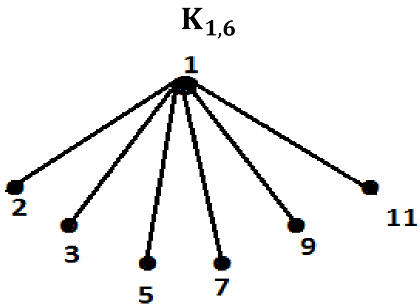
Let the central vertex of the star be u. The other vertices are $v_1, v_2, \ldots, v_n$ respectively. Now we consider the following cases.

Case (i) 2< $n \leq 8$. Assign u = 1, $v_1 = 2$ and $v_i = 2i - 1$, $2 \leq i \leq 8$.

Then the labeling pattern is given below







$K_{1,8}$

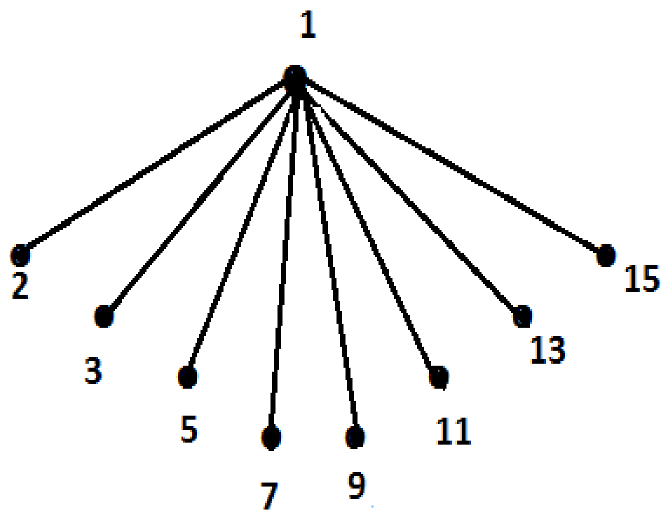
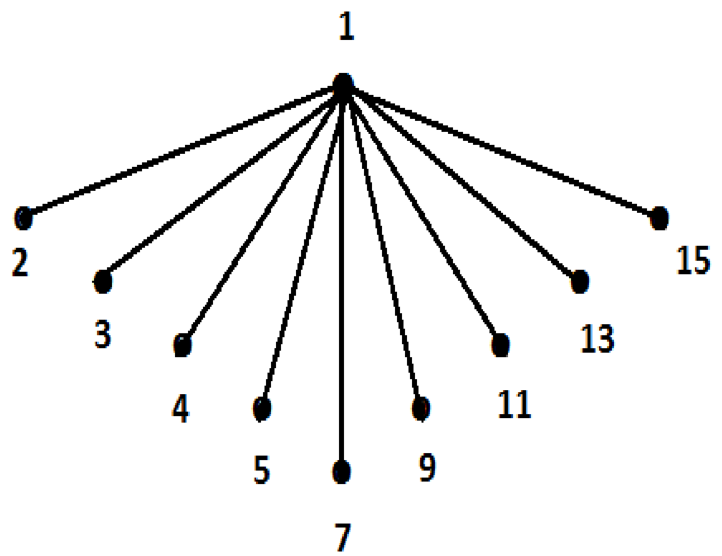


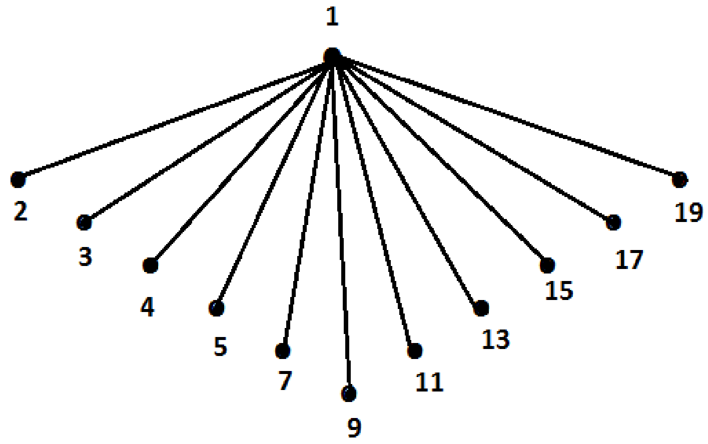
Figure:7

Case (ii) $9 \leq n \leq 15$. Assign u = 1, $v_1 = 2, v_2 = 3, \ v_3 = 4$ and $v_i = 2i - 3$, $4 \leq i \leq 15$. Then the labeling pattern is given below.

$K_{1,9}$



$K_{1,11}$



$K_{1,15}$

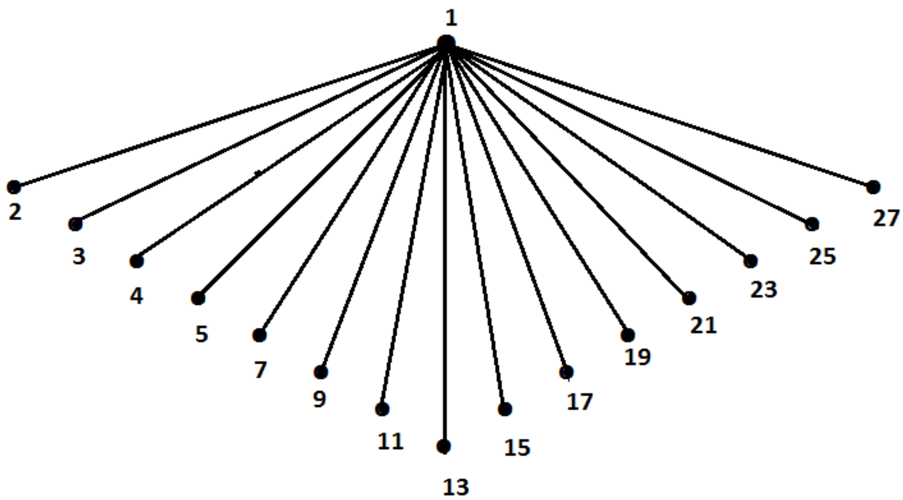


Figure:8

Clearly this labeling pattern is Stolarsky-3 Mean graph.

Case (iii) n>15

Let the label of the vertices u =1 ,$v_1 = 2$, $v_2 = 3$, $v_3 = 4$, $v_i = 2i - 3$, 4$\leq i \leq n$

$K_{1,n}$

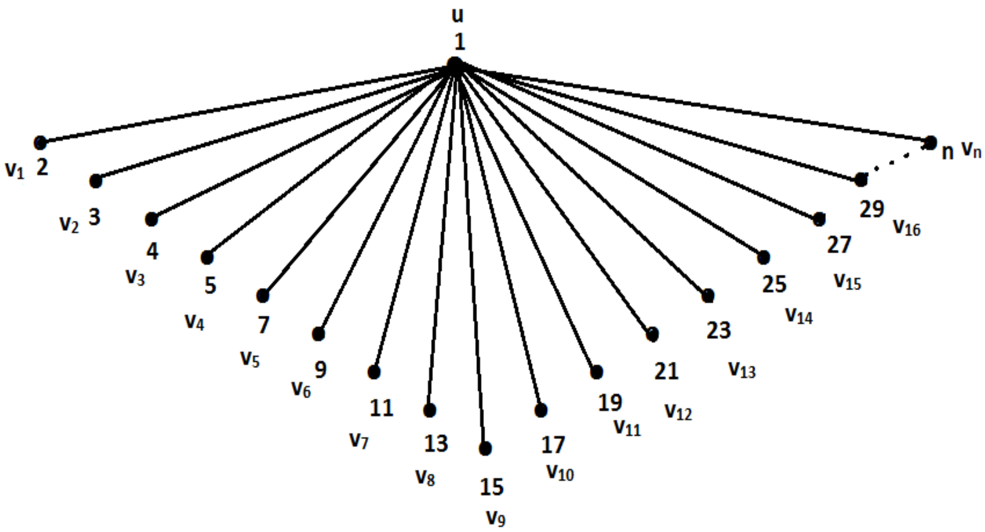


Figure:9

Here the edge labels of uv_{15} is 15 and uv_{16} is 17. The number 16 missing which is not possible. From case (i), case(ii) and case(iii) ,we conclude that $K_{1,n}$ is a Stolarsky-3 Mean graph if n ≤ 15.

3. Conclusion

In this paper we introduced the concept of Stolarsky-3 Mean labeling and studied the stolarsky-3 Mean labeling behavior of some standard graphs. The authors are of the opinion that the study of Stolarsky-3 Mean labeling behavior of graph obtained from standard graphs using the graph operation shall be quite interesting and also will lead to new results.

4. Acknowledgements

The authors are thankful to the referee for their valuable comments and suggestions.

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